2-

$$f(t) = a\frac{\sin(\pi a t)}{a\pi t} = a\,\mathrm{sinc}(at) \xleftrightarrow{\mathcal{F}} \Pi(f/a)$$

$$g(t) = b\,\mathrm{sinc}(bt) \xleftrightarrow{\mathcal{F}} \Pi(f/b)$$

using parseval's Theorem

$$E = \int_{-\infty}^{\infty} \left(\frac{df}{dt} * \frac{dg}{dt}\right)^2 dt = \int_{-\infty}^{\infty} \left(\mathcal{F}\left\{\frac{df}{dt}\right\} \mathcal{F}\left\{\frac{dg}{dt}\right\}\right)^2 df$$

$$= \int_{-\infty}^{\infty} \left(j2\pi f\, \Pi\left(\frac{f}{a}\right) \cdot j2\pi f\, \Pi(f/b)\right)^2 df$$

$$= \int_{-b/2}^{b/2} 16\pi^4 f^4\, df \qquad (\text{assuming } b<a)$$

$$= \frac{16\pi^4}{5} f^5 \Big|_{-b/2}^{b/2} = \frac{\pi^4 b^5}{5}$$

3-

$$f(t) = \Pi(t/a) \xleftrightarrow{\mathcal{F}} a\,\mathrm{sinc}(af)$$
$$g(t) = \Pi(t/b) \xleftrightarrow{\mathcal{F}} b\,\mathrm{sinc}(bf)$$

(a) $$t\,g(t) \xleftrightarrow{\mathcal{F}} \left(\frac{1}{2\pi}\right)\frac{d}{df}\left\{\cancel{b}\,\frac{\sin \pi bf}{\pi \cancel{b} f}\right\} = \frac{j}{2\pi^2}\frac{d}{df}\left(\frac{\sin(\pi bf)}{f}\right)$$

$$= \frac{j}{2\pi^2}\,\frac{f\pi b\cos(\pi bf) - \sin(\pi bf)}{f^2}$$

$$\therefore\ f(t) * t\,g(t) \xleftrightarrow{\mathcal{F}} a\frac{\sin(\pi af)}{\pi af} \cdot \frac{j}{2\pi^2}\,\frac{f\pi b\cos(\pi bf) - \sin(\pi bf)}{f^2}$$

$$= \frac{j}{2\pi^3 f^3}\left(\left[\pi bf\cos(\pi bf) - \sin(\pi bf)\right]\sin(\pi af)\right)$$

(b) $$f(t) * g(t) \xleftrightarrow{\mathcal{F}} \frac{\sin(\pi af)\,\sin(\pi bf)}{\pi^2 f^2}$$

$$t\,(f(t) * g(t)) \xleftrightarrow{\mathcal{F}} \frac{1}{\pi^2}\left(\frac{j}{2\pi}\right)\frac{d}{df}\left(\frac{\sin(\pi af)\cdot\sin(\pi bf)}{f^2}\right)$$

$$= \frac{1}{2\pi^3 f^4}\Big(f^2\left[\sin(\pi af)\cdot \pi b\cos(\pi bf) + \pi a\cos(\pi af)\cdot\sin(\pi bf)\right] - 2f\left[\sin(\pi af)\,\sin(b\pi f)\right]\Big)$$

$$= \frac{j}{2\pi^3 f^3}\left[f\left[(\pi b\sin(\pi af)\cos(\pi bf) + \pi a\cos(\pi af)\right] - \left[2\sin(\pi af)\sin(\pi bf)\right]\right]$$