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ECE 521 Experiment # 4: On The Discrete Fourier Transform (DFT)

(Issued Thursday 4/15, Due Tuesday 5/5)

Purpose: The purpose of this experiment is the following:

1. Quantify the effects of windowing and spectral sampling associated with the DFT;
2. Evaluate an estimate of the power spectrum of a signal.
3. Study linear convolution using the FFT algorithm
4. Experiment with elementary speech processing techniques.

Part 1: DFT Analysis of Sinusoidal Signals

This part is described in the textbook Chapter 10 and Chapter 7 of the Class Notes. Its main purpose is to quantify the effects of windowing (leakage) and spectral sampling (that may result in dislocation of the spectral peaks). These phenomena are essentially unavoidable and one should always be aware of their effects on the signal spectrum. A typical system for DFT analysis of continuous-time signals is shown below:

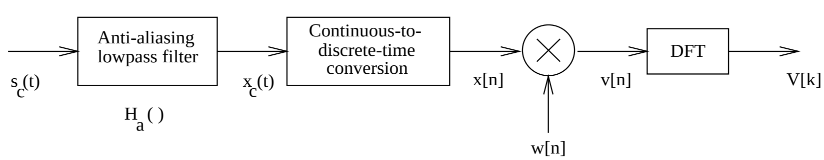

Fig. 4.1. A block diagram of a signal processing system that uses the DFT.

As shown on Fig. 4.1, The signal $s_c(t)$, the measured (actual) signal, is passed through a low-pass filter with impulse response $h_a(t)$ to limit the bandwidth (this filter is usually an analog filter). The resulting bandlimited signal $x_c(t) = s_c(t) * h_a(t)$ is sampled and quantized (using an A/D converter) to obtain the sequence x[n] which can be very long. In practice, we deal with finite number of samples, therefore, we obtain a shorter sequence v[n] by shopping off a portion of x[n] by a window function. In other words, v[n] = x[n] w[n], where w[n] is a window function. Now we use the DFT of v[n], V[k], to give us some information about the frequency spectrum of the original signal $s_c(t)$.

The digital signal processing system as described above is subject to various sources of errors. Distortion of the original spectrum $S_c(f)$ can result from the low-pass filter $h_a(t)$ (e.g., inappropriate bandwidth). Errors also can result from the A/D operation (e.g., inappropriate sampling and/or inaccurate quantization), and due to limiting the number of samples in the sequence v[n] (the so-called finite length or windowing effect).

We will ignore the errors due to $h_a(t)$ and the A/D converter, and concentrate on the errors introduced by w[n] (windowing effect) and the intrinsic limitations of the DFT (note that V[k] is a sampled version of $V(f_d)$). In order to quantify these effects, we will use sinusoidal signals so we will know exactly the location of the spectral peaks and their amplitudes. We would like to drive home the importance of proper interpretation of the results of the DFT. Consider the signal

$$s_c(t) = A_1 \cos(2\pi f_1 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2), \quad (1)$$

where A_1 and A_2 are constants (amplitudes), f_1 and f_2 are the sinusoidal frequencies, and θ_1 and θ_2 are the phase shifts.

I. Effect of windowing

Let $f_1 = 5/3$kHz and $f_2 = 10/3$kHz, $\theta_1 = \theta_2 = 0^\circ$, and $A_1 = 1$ and $A_2 = 0.75$. Assume ideal $h_a(t)$ and sample $x_c(t)$ at a rate $f_s = 1/T = 10$kHz. Ignore all quantization errors (i.e., the sequence x[n] will be equal to $x_c(nT)$).

(i) Consider a rectangular window of length L = 64; i.e., $w[l] = 1$, $0 \le l \le 63$. Obtain an expression for $W(f_d)$ and $V(f_d)$ (check your results with textbook Eq. 10.10 and 10.11) and plot one period ($-0.5 \le f_d \le 0.5$) of $W(f_d)$ and $V(f_d)$ (check your results with textbook Fig. 10.3).

(ii) Fix L and f_s as above while changing f_1 and f_2 around the above values, and observe the leakage effect.

(iii) For the same frequencies in (ii), evaluate and plot 64-points of the DFT sequences W[k] and V[k] using an FFT algorithm (you may use the "fft.c" program on the class website, or you may use Matlab). Properly scale your plots to correspond to those in (i)-(ii). Comment on the correspondence between the sequence Fourier Transform and the DFT.

II. Sampling the DFT

Since $V[k] = V(f_d)$, $f_d = k/N$ a situation can evolve in which the peaks in V[k] will not correspond to the peaks in $V(f_d)$. We will study such a possibility and show that by increasing the length of the DFT we may be capable of getting closer to the exact peak location.

(i) Consider the signal in Eq. 1 above with $f_1 = 1/16$kHz, $f_2 = 1/8$kHz, $\theta_1 = \theta_2 = 0^\circ$, $A_1 = 1$ and $A_2 = 0.75$, and use $f_s = 1$kHz. For a rectangular window of length 64, obtain and plot $V(f_d)$ and V[k] as before. Check your results with Fig. 10.6. Observe that only two spectral peaks are present in V[k]. What is the mathematical interpretation of this behavior of V[k]?

(ii) Repeat (i) with DFT of lengths 128 and 256. Observe the presence of more peaks and the leakage effect.

Part 2. Power Spectral Estimation

The purpose of this part is to get experience with some of the issues of power spectral estimation. Write a program to perform the classical power spectral estimation using the periodogram method; i.e., for a sequence v[n] of length N, the power spectrum $P_v(f_d)$ is estimated by

$$\hat{P}_v(f_d) = \frac{1}{N}\left(\left|\sum_{n=0}^{N-1} v[n]\, e^{-j2\pi f_d n}\right|\right)^2. \qquad (2)$$

Write a program to calculate a sampled version of $\hat{P}_v(f_d)$, $\hat{P}_v[k]$, using the DFT. Note that $\hat{P}_v(f_d)$ is suppose to convey the power spectrum of the original signal $s_c(t)$ in Fig. 1.

(i) Test the program to estimate the power spectrum of the signal

$$s_c(t) = 10\cos(2\pi 0.2t) + 20\cos(2\pi t) + 30\cos(2\pi 1.2t) + \alpha\nu(t). \qquad (3)$$

where, $\nu(t)$ is a zero mean Gaussian noise signal with variance 0.5 and α is a constant that takes the values between 0 and 10. Generate a 1024 sequence x[n] (recall the sampling theorem!) from $s_c(t)$, assuming ideal $h_a(t)$ and no A/D errors. You may use the program "noise.c" in the class website, or use MatLab, to generate the Gaussian noise samples. Plot x [n] vs n in units of seconds for various α.

(ii) Using the windows in Part 1, Lab 3, with length 256, obtain the sequence v[n] = x[n] w[n] for various windows and evaluate $\hat{P}_v(f_d)$ and plot one period for $-0.5 \le f_d \le 0.5$. Plot the results of all windows on the same scale.

(iii) Repeat (ii) for various windows to evaluate $\hat{P}_v[k]$ using the DFT. Plot a properly scaled version of $\hat{P}_v[k]$ for all windows on the same scale. Knowing the exact locations of the spectral peaks, discuss the results of each window.

(iv) Repeat (ii)-(iii) using the signal z[n] in the class website, knowing that $f_s = 25$Hz.

Part 3. Convolution in the Frequency Domain

The purpose of this part is to use the FFT algorithm for linear convolution. Recall that the FFT algorithm is a quick implementation of the DFT equation and that the DFT is a sampled version of the sequence Fourier transform. Recall also the intrinsic periodicity in the DFT and that all operations on the DFT corresponds to discrete Fourier series operations observed over one period. These points must not be overlooked when we use the FFT algorithm to calculate the linear convolution.

Write a program that will take as its input two sequences x[n] and h[n] of lengths N_1 and N_2, respectively and calculates the linear convolution of the two sequences $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$ using the FFT algorithm. Test your program using known sequences to check your results. Note: you may use the FFT on MatLab.

Let the sequence x[n] be the 512-point sequence in the file "z" in the class website that was obtained from the wrist impedance signal sampled at 25Hz. The wrist impedance is known to provide a measure for the heart rate (HR) and the respiration rate (RR). Using the filters designed in Part 1, we like to separate the signals corresponding to the HR and the RR. Using common sense, select the required filter parameters to achieve this purpose. Plot the original sequence, the HR sequence, and the RR sequence on the same scale (three separate graphs under each other with same scale on the abscissa). Plot the filters' response you have used.

Part 4. Elementary Speech Processing

The purpose of this part is to obtain some basic understanding of the frequency contents of speech signals. Generate a speech signal of the utterance *"I took ECE520 Spring 04 and I learned a lot!"* using a sampling rate of 16 kHz. You may use any microphone and a PC.

(a) Plot the analog signal (amplitude vs time in seconds).

(b) Using 512-point segments calculate the power spectrum using the techniques of Part 2 and the Kaiser window.

(c) Obtain a 3-D plot of the magnitude spectrum vs time. Properly label your axes.

(d) Obtain an 8-bit character representation (image) of the results in (c) and print the image properly labeled. This image is known as the spectrogram.

(e) Using the results in (c) and (d) identify the wideband and narrowband regions of the spectrogram.

(f) Using the bands in (e) design an FIR filter (Lab 3) to isolate these bands by linear convolution (Part 3). Play back your results - How does it sound?

Notes:

1) Write a neat report. All figures should have meaningful captions, properly labeled, and referred to in the text.

2) This Lab has 20 extra points than other Labs.