② Let $x_1(t) = \begin{cases} 1 & |t| \leq T \\ 0 & \text{elsewhere} \end{cases}$

$$x_2(t) = \begin{cases} t & |t| \leq 2T \\ 0 & \text{elsewhere} \end{cases}$$

(a) Convolution:-

$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(t-\tau)\, x_2(\tau)\, d\tau$$

$\Rightarrow$ for $T + t \leq -2T \rightarrow t \leq -3T$

$\boxed{y(t) = 0}$ for $t \leq -3T$

$\Rightarrow$ for $-2T \leq T+t \leq 0 \longrightarrow -3T \leq t \leq -T$

$$y(t) = \int_{-2T}^{t+T} \tau \, d\tau \qquad -3T \leq t \leq -T$$

$$\boxed{y(t) = \frac{1}{2}\left[t^2 + 2Tt - 3T^2\right]} \qquad -3T \leq t < -T$$

$\Rightarrow$ for $0 \leq T+t \leq 2T \longrightarrow -T \leq t \leq T$

$$y(t) = \int_{t-T}^{t+T} \tau \, d\tau \qquad -T \leq t \leq T$$

$$\boxed{y(t) = 2Tt} \qquad \text{for } -T \leq t \leq T$$

$\Rightarrow$ for $2T \leq t+T \leq 4T \longrightarrow T \leq t \leq 3T$

$$y(t) = \int_{t-T}^{2T} \tau \, d\tau \qquad T \leq t \leq 3T$$

$$\boxed{y(t) = \frac{1}{2}\left[3T^2 + 2Tt - t^2\right]}$$

$\Rightarrow$ for $t \geq 3T \quad y(t) = 0$

$$\therefore \; y(t) = \begin{cases} 0 & \longrightarrow \quad t \leq -3T \\ \frac{1}{2}\left[t^2 + 2Tt - 3T^2\right] & \longrightarrow \quad -3T \leq t \leq -T \\ 2Tt & \longrightarrow \quad -T \leq t \leq T \\ \frac{1}{2}\left[3T^2 + 2Tt - t^2\right] & \longrightarrow \quad T \leq t \leq 3T \\ 0 & \longrightarrow \quad t \geq 3T \end{cases}$$

(b) autocorrelation :-

$$R_{x_1}(t) = \int_{-\infty}^{\infty} x_1(t+\tau)\, x_1(\tau)\, d\tau = \int_{-\infty}^{\infty} x_1(\tau)\, x_1(\tau - t)\, d\tau$$

$\Rightarrow$ for $t + T \leq -T \longrightarrow t \leq -2T$ $\qquad R_{x_1}(t) = 0$

$\Rightarrow$ for $-T \leq t + T \leq T \longrightarrow -2T \leq t \leq 0$

$$R_{x_1}(t) = \int_{-T}^{t+T} 1\, d\tau = t + 2T$$

$\Rightarrow$ for $T \leq t \leq 3T \longrightarrow 0 \leq t \leq 2T$

$$R_{x_1}(t) = \int_{t-T}^{T} 1\, d\tau = 2T - t$$

$\Rightarrow$ for $t \geq 2T$ $\qquad R_{x_1}(t) = 0$

$$\therefore \quad R_{x_1}(t) = \begin{cases} t + 2T & -2T \leq t \leq 0 \\ 2T - t & 0 \leq t \leq 2T \\ 0 & \text{elsewhere} \end{cases}$$

(c) Cross-Correlation

$$R_{x_2 x_1}(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t+\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau - t)\, x_2(\tau)\, d\tau$$

$\because$ $x_1(t)$ is even $\therefore$ $x_1(\tau - t) = x_1(t - \tau)$

$$\therefore \quad R_{x_2 x_1}(t) = \int_{-\infty}^{\infty} x_1(t - \tau)\, x_2(\tau)\, d\tau$$

$\therefore$ in this case $R_{x_2 x_1}(t)$ is identical to $y(t) = x_1(t) * x_2(t)$