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ECE521 Experiment # 4: On The Discrete Fourier Transform (DFT)

(Issued Tuesday 7/17, Due Tuesday 7/24)

The purpose of this experiment is to quantify the effects of windowing and spectral sampling associated with the DFT.

1 DFT Analysis of Sinusoidal Signals

This part is described in the textbook Chap. 11, pp. 696-713. Its main purpose is to quantify the effects of windowing (leakage) and spectral sampling (that may result in dislocation of the spectral peaks). These phenomena are essentially unavoidable and one should always be aware of their effects on the signal spectrum. A typical system for DFT analysis of continuous-time signals is shown in Fig. 11.1 of the textbook.

As shown on Fig. 1, The signal s_c(t), the measured (actual) signal, is passed through a low-pass filter with impulse respons h_a(t) to limit the bandwidth (this filter is usually an analog filter). The resulting bandlimited signal x_c(t) = s_c(t) * h_a(t) is sampled and quantized (using an A/D converter) to obtain the sequence x[n] which can be very long. In practice, we deal with finite number of samples, therefore, we obtain a shorter sequence v[n] by shopping off a portion of x[n] by a window function. In other words, v[n] = x[n] w[n], where w[n] is a window function. Now we use the DFT of v[n], V[k], to give us some information about the frequency spectrum of the original signal s_c(t).

The digital signal processing system as described above is subject to various sources of errors. Distortion of the original spectrum S_c(f) can result from the low-pass filter h_a(t) (e.g., inappropriate bandwidth). Errors also can result from the A/D operation (e.g., inappropriate sampling and/or inaccurate quantization), and due to limiting the number of samples in the sequence v[n] (the so-called finite length or windowing effect).

We will ignore the errors due to h_a(t) and the A/D converter, and concentrate on the errors introduced by w[n] (windowing effect) and the intrinsic limitations of the DFT (note that V[k] is a sampled version of V(f_d)). In order to quantify these effects, we will use sinusoidal signals so we will know exactly the location of the spectral peaks and their amplitudes. We would like to drive home the importance of proper interpretation of the results of the DFT. Consider the signal

$$s_c(t) = A_1\ cos(2\pi f_1\ t + \theta_1) + A_2\ cos(2\pi f_2\ t + \theta_2), \tag{1}$$

where A_1 and A_2 are constants (amplitudes), f_1 and f_2 are the sinusoidal frequencies, and θ_1 and θ_2 are the phase shifts.

1.1 Effect of windowing

Let f_1 = 5/3kHz and f_2 = 10/3kHz, $\theta_1 = \theta_2 = 0$, and $A_1 = 1$ and $A_2 = 0.75$. Assume ideal h_a(t) and sample x_c(t) at a rate f_s = 1/T = 10kHz. Ignore all quantization errors (i.e., the sequence x[n] will be equal to $x_c(nT)$).

1. Consider a rectangular window of length L= 64; i.e., w[l] = 1, $0 \leq l \leq 63$. Obtain an expression for W(f_d) and V(f_d) (check your results with textbook Eq. 11.10 and 11.11) and plot one period ($-0.5 \leq f_d \leq 0.5$) of W(f_d) and V(f_d) (check your results with textbook Fig. 11.3).

2. Fix L and f_s as above while changing f_1 and f_2 around the above values, and observe the leakage effect.

3. For the same frequencies in (2), evaluate and plot 64-points of the DFT sequences W[k] and V[k] using an FFT algorithm. Properly scale your plots to correspond to those in (1)-(2). Comment on the correspondence between the sequence Fourier Transform and the DFT.

1.2 Sampling the DFT

Since V[k] = V(f_d), f_d = k/N a situation can evolve in which the peaks in V[k] will not correspond to the peaks in V(f_d). We will study such a possibility and show that by increasing the length of the DFT we may be capable of getting closer to the exact peak location.

1. Consider the signal in Eq. 1 above with $f_1 = 1/16$kHz, $f_2 = 1/8$kHz, $\theta_1 = \theta_2 = 0$, $A_1 = 1$ and $A_2 = 0.75$, and use $f_s = 1$kHz. For a rectangular window of length 64, obtain and plot V(f_d) and V[k] as before. Check your results with Fig. 11.6. Observe that only two spectral peaks are present in V[k]. What is the mathematical interpretation of this behavior of V[k]?

2. Repeat (1) with DFT of lengths 128 and 256. Observe the presence of more peaks and the leakage effect.

Note: *Write a neat report. All figures should have meaningful captions, properly labeled, and referred to in the text. Refrain from using any handwritten symbols, equations, etc. in your report, use a word-processor*