ECE643: Introduction to Biomedical Computing

Lab Assignment to MRI Reconstruction Using K-spaces

This lab assignment concerns partial k-space reconstruction methods.

1. Motivation for Partial k-Space Reconstruction

In theory, most MRI images depict the spin density as a function of position, and hence should be real valued. If this were true, then by the symmetry of the Fourier transform, only half of the spatial-frequency data will need to be collected.

Since real functions have conjugate symmetry in spatial frequency space, the uncollected data could be synthesized by reflecting conjugate data across the origin. Unfortunately, there are many sources of phase errors that cause the real-valued assumption to be violated. These include variations in the resonance frequency, flow, and motion. As a result, partial k-space reconstructions always require some type of phase correction, to correct for these sources of incidental phase variation. This allows real images to be reconstructed.

An example of a gradient-recalled axial head image shown in Figure 1 illustrates the problem. The magnitude reconstruction of a full k-space acquisition is shown in Figure 1a and, the phase in Figure 1b. The linear component of the phase has been corrected, leaving only the non-linear components. The absolute values of the real and imaginary components are shown in Figure 1c and d. Clearly, a significant amount of phase correction is required before the conjugate phase symmetry can be exploited.

The reason for this phase is that the precession frequency varies across the head. The image phase is approximately

$$\phi(x, y) = \omega(x, y)T_E \tag{1}$$

where, $\omega(x, y)$ is the local resonant frequency in *rad/s*, and T_E is the echo time.

These changes in frequency vary slowly with spatial position, and can in theory be calibrated out of the system with proper shimming. More fundamental, and more problematic, is the variation in frequency due to the magnetic susceptibility difference between tissue and air. At air-tissue interfaces it is common to see local frequency shifts of several parts-per-million (ppm). These are seen in these images in the brain tissue adjacent to the ears, which is directly above the auditory canal, and around the nasal passages. An additional problem is that these shifts can occur over relatively short distances, and this governs the amount of resolution required for phase compensation, the amount of coverage in k-space that will be required, and ultimately how \partial" a partial k-space acquisition can be.

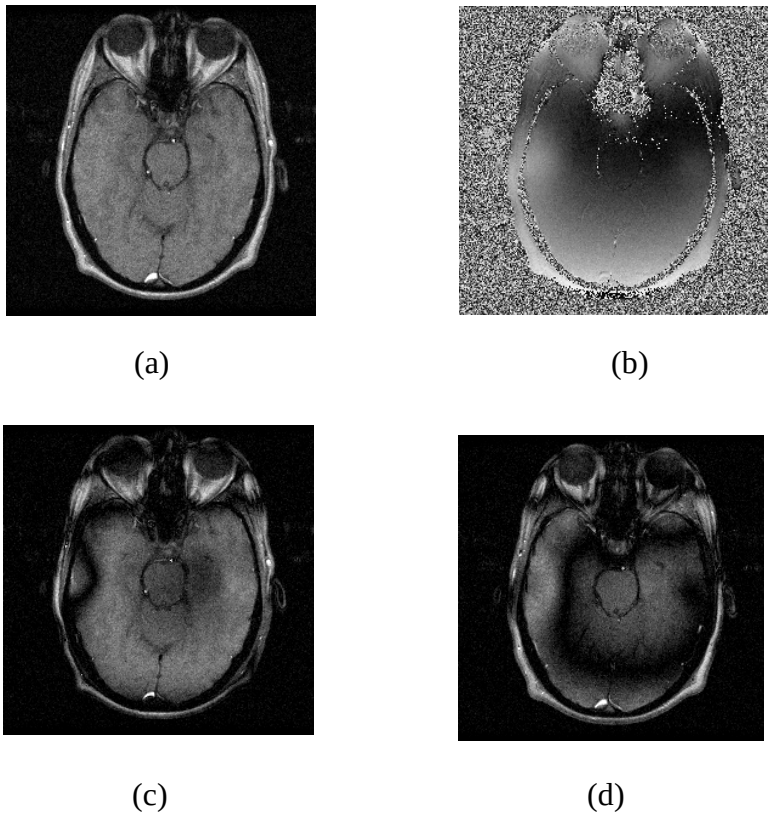


Figure 1. Axial gradient echo image acquired at 0.5T with an echo time of 13.8 ms. At this time water and fat are re-phased. The linear shim terms have been corrected, leaving the non-linear components due to susceptibility shifts. (a) magnitude image, (b) phase image, (c) absolute of real part, and (d) absolute of imaginary part

2. 2D FT (Fourier Transform) Applications

There are two general applications of 2DFT imaging where it is desirable to collect only a fraction of the full k-space data. The first is for reducing scan time by reducing the number of acquisitions that are required to construct an image of a given resolution. This is illustrated in Figure 2. Slightly more than half of the complete k-space data is collected, allowing the scan time to be reduced by almost a factor of two.

The second application is for reducing echo times. Here the area of the readout de-phasor gradient is reduced so that the echo comes earlier in the readout window as is shown in Figure 3. This can be important for reducing flow dependent de-phasing, and through plane susceptibility-induced signal loss. This case is illustrated in Figure 4.

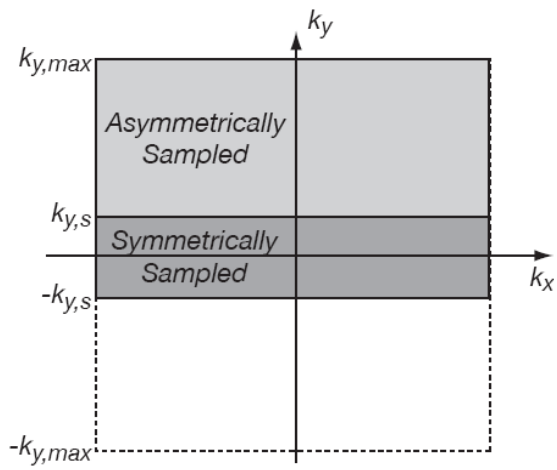


Figure 2. Partial k-space acquisition for reducing scan time by reducing the number of phase encodes required.

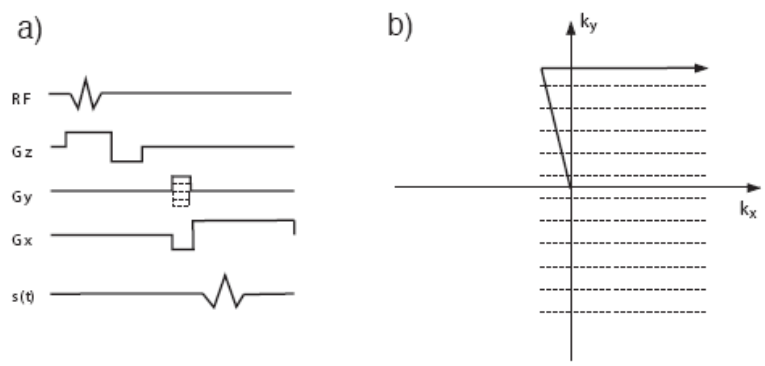


Figure 3. Pulse sequence with a reduced echo time, in a partial echo readout.

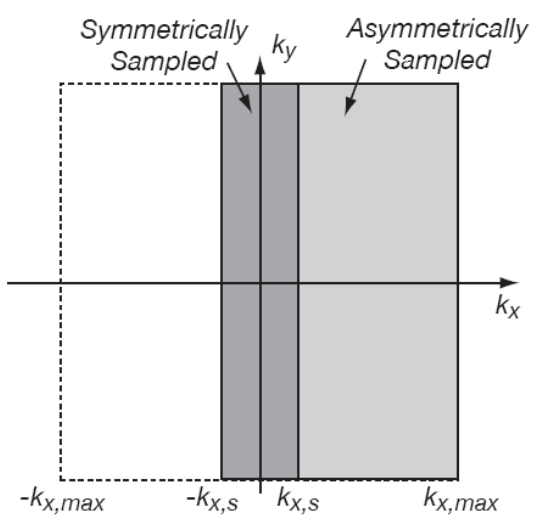


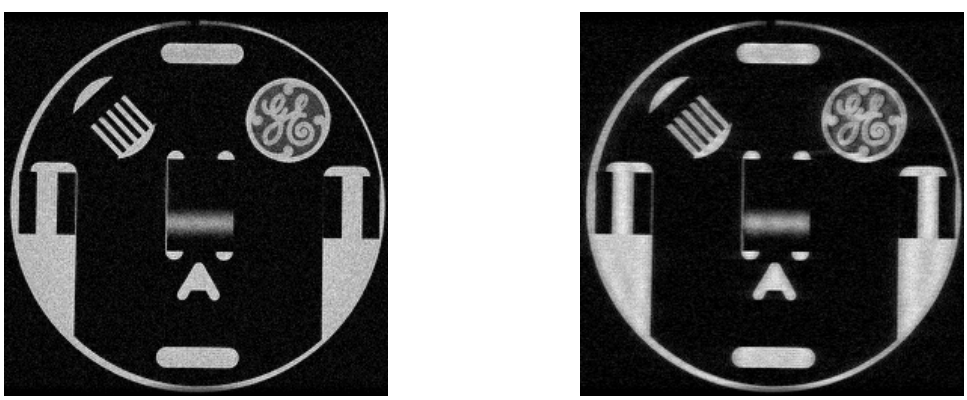
Figure 4. Partial k-space acquisition for reducing echo times by collecting only a fraction of the full echo.

3. Direct Partial k-Space Reconstruction

3.1 Trivial Reconstruction by Zero Padding

The simplest way to reconstruct a partial k-space data set is to simply fill the uncollected data (phase-encodes or readout samples) with zeroes. Then, perform the 2DFT and display the magnitude. This works acceptably if the collected k-space fraction is close to 1, and works poorly as this fraction approaches 0.5. This is illustrated in Figure 5 for a k-space fraction of 9/16ths. The reconstruction of the full k-space data is shown in (a), and the reconstruction of the zero-padded partial k-space data in (b). The result is significant blurring in the phase-encode direction. Clearly this is unacceptable, and this motivates the search for other solutions. The reason for the blurring can be identified by considering the data set to be the product of a full k-space data set multiplied by a weighting function.

In this case an offset step function, where the offset corresponds to the k-space fraction. This will be denoted $W(k_y)$, and is illustrated in Figure 6. The inverse Fourier transform of this function is the impulse response that produces the blurring.
If we look at the real component, we see a sharp impulse at the desired resolution plus a broader component that corresponds to the width of the symmetrically acquired data. There is also a significant undesired imaginary component.



(b)

Figure 5. Comparison of a reconstruction of a full k-space data (a), and a trivial partial k-space reconstruction (b) of the same data set where only 144 of 256 phase encodes have been used, and the remaining 112 have been replaced by zeros. Note the significant blurring in the phase-encode (left-right) direction.

3.2 Phase Correction and Conjugate Synthesis

In order to correct for the blurring from the trivial reconstruction we need to fill in the missing uncollected data.

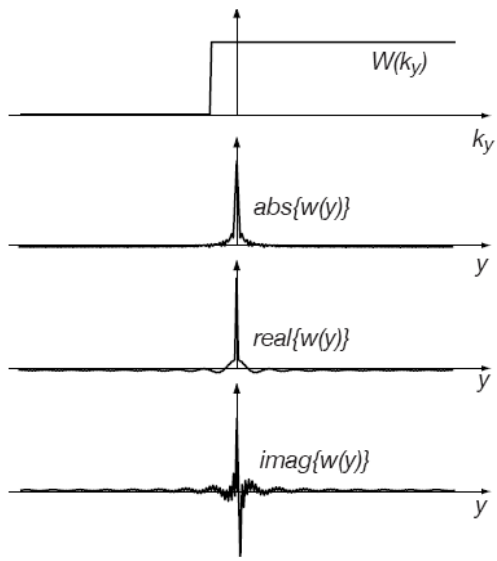


Figure 6. A k-space weighting function $W(k_y)$that truncates a full k-space data set into a partial k-space data set. In this case the k-space fraction is 9/16ths, as in Fig. 5. The blurring evident is Fig. 5 is due to the convolution of the inverse transform of $W(k_y)$, which is plotted here in absolute value, and real and imaginary components.

From Figure 1 it is clear that in general phase correction must be applied before the k-space symmetry can be exploited to synthesize the missing data. In order to do the phase correction, we will use the narrow strip of data for which we have symmetric coverage. The phase of this low resolution image is then used to phase correct the partial k-space data. After inverse transforming the phase correction, the image reconstructed from the partial k-space data is transformed back to the spatial frequency domain, where the data corresponding to the missing data is synthesized by conjugate symmetry,

$$M(k_x, k_y) = M^*(-k_x, -k_y) \tag{2}$$

This process is illustrated in Figure 7. The partial k-space data is $M_{pk}(k_x, k_y)$, $M_s(k_x, k_y)$ is the narrow strip of symmetric data, and $m_{pk}(x, y)$ and $m_s(x; y)$ are the corresponding images produced by a inverse Fourier transform. The phase correction function is a unit amplitude image with a phase that is the conjugate of $m_x(x; y)$,

$$p^*(x, y) = e^{-i\angle(m_s(x,y))} \tag{3}$$

This approach also has a little problem due to the effects of the phase compensation step near the boundary of the acquired data. The multiplication by the phase compensation function in the image domain is a convolution in the frequency domain, and the size of this convolution function can be significant. The fact that the convolution is operating on zero data for the uncollected phase encodes produces errors near the boundary.

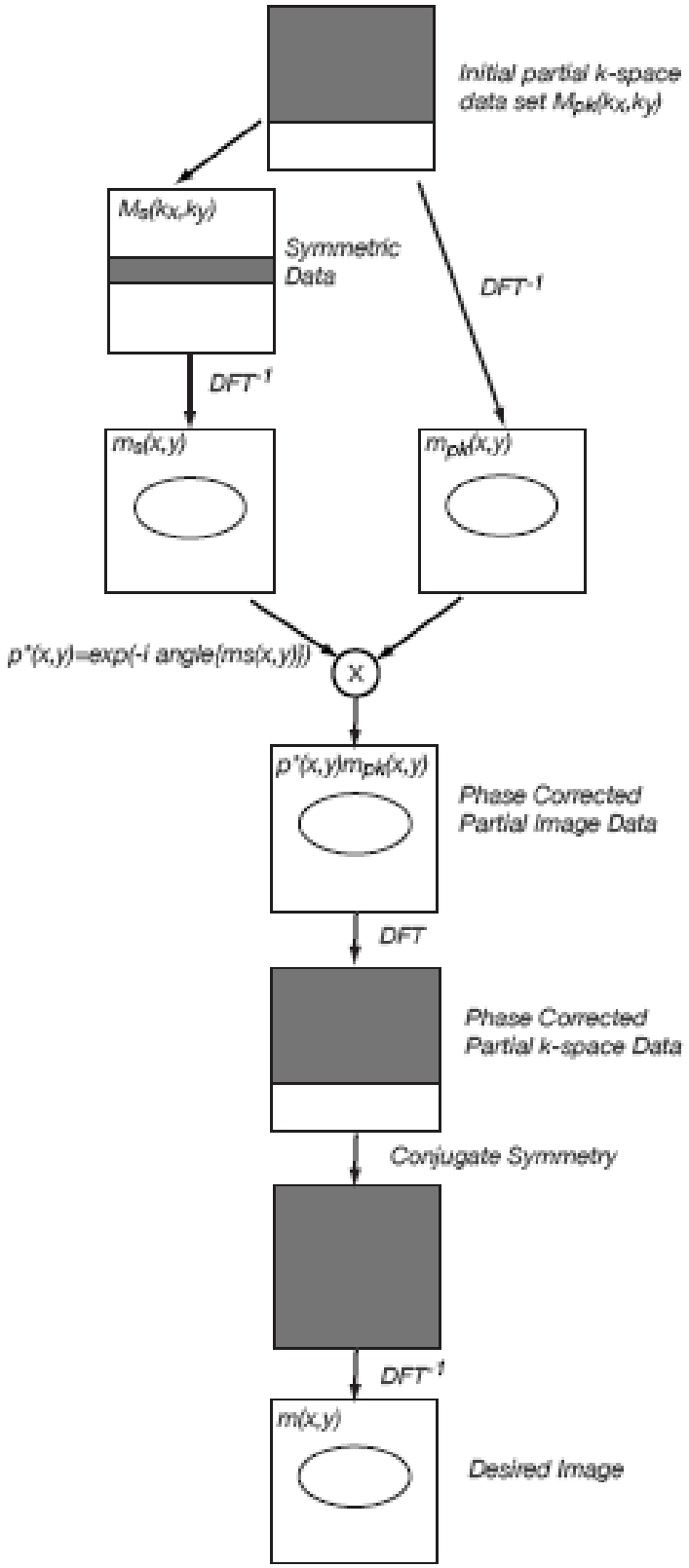


Figure 7. The phase correction and conjugate synthesis algorithm.

4. Data Files

The MR data files are in
http://www.cvip.louisville.edu/wwwcvip/frames/mainFrames/Education.htm

under ECE643 folder.

Most of the data sets were acquired on a 0.5T interventional scanner using a head coil for transmit and receive. The files include:

Spin Echo Data Sets:

se_t1_sag_data.mat: Sagittal T1-weighted midline shot of the head of a normal volunteer. CSF is dark due it its very long T1, white matter is brighter than gray matter. Fat is very bright due to it's short T1.

fse_t1_ax_ data.mat: Axial T1-weighted shot of the head of a normal volunteer at the level of the eyes. Same contrast as the previous shot. This was acquired with a fast-spin-echo pulse sequence.

Fse_t2_ax_data.mat Axial T2-weighted shot of the head of a normal volunteer at the level of the eyes. Gray matter is brighter than gray matter, CSF is bright due to its long T2. Fat is bright due to the suppression of J-coupling along the lipid molecules by the rapid refocusing of the fast-spin-echo pulse sequence, which lengthens the T2 significantly.

All are 256*256 data matrices.

4.2 Some Hints on the dataset:

To generate a 3/4th partial k-space data set from a full data matrix d, one alternative is

```
>> dp = zeros(nro,npe);
>> dp(:,1:(npe*3/4)); = d(:,1:(npe*3/4));
```

where nro is the number of readout samples, and npe is the number of phase encodes (both 256 so far), and we have chosen the first 3/4ths of the phase encodes. Similarly, a partial echo data set is generated by

```
>> dp = zeros(nro,npe);
>> dp((nro/4+1):nro,:); = d((nro/4+1):nro,:);
```

where the first part of the echo has not been collected, which is the usual case.

5. Lab Assignment:

5.1. Conventional 2DFT Reconstruction (for both master and Ph.D students)

Choose one of the spin echo data sets, and answer the following questions:

Is the largest k-space sample at the origin? If not, which direction is it shifted? What does that imply about the acquisition?

5.2 Phase Correction and Conjugate Synthesis Algorithm (Ph.D Students Only, bonus for master students)

Choose one of the spin echo data sets.

We will assume a 9/16th k-space acquisition with 256 phase encodes. This means there are 112 unacquired phase encode, 32 symmetrically sampled phase encodes, and 112 phase encodes that are only sampled for positive spatial frequencies.

Use the Phase Correction and Conjugate Synthesis Algorithm as shown in Figure 7 to do the reconstruction.

References:

ECE369: Medical Image Reconstruction John Pauly, Stanford University, Fall, 2007.